Q1

1

The first three terms of a geometric sequence are given by $x + 12$, $3x$, and x^2 respectively, where x is a non-zero real number.

Find the value of the 102nd term in the sequence.

[5]

$r = \frac{3x}{x+12} = \frac{x^2}{3x}$

$9x^2 = x^2(x+12)$

$9x^2 = x^3 + 12x^2$

$0 = x^3 + 3x^2$

$0 = x^2(x+3)$

$x = \cancel{0}, -3$ x is non-zero $\therefore x = -3$

Sub $x = -3$ into eqns for a and r.

$a = x + 12 = -3 + 12 = 9$

$r = \frac{3x}{x+12} = \frac{3(-3)}{-3+12} = \frac{-9}{9} = -1$

So the sequence is: $9, -9, 9, -9, \ldots$

102 is even, and all even terms have a value of -9 $\therefore$ 102nd term: $\boxed{-9}$

(we can also $u_n = ar^{n-1}$ to find u_{102})

Q2

2

The sum of the first three terms in a geometric series is 8.75.
The sum of the first six terms in the same series is 13.23.

Find the common ratio, r, of the series.

[4]

$S_3 = 8.75$ $S_6 = 13.23$ $S_n = \frac{a(1-r^n)}{1-r}$

$8.75 = \frac{a(1-r^3)}{1-r}$ $13.23 = \frac{a(1-r^6)}{1-r}$

difference of 2 squares!

$\frac{S_6}{S_3}$: $\frac{13.23}{8.75} = \frac{\frac{a(1-r^6)}{1-r}}{\frac{a(1-r^3)}{1-r}} = \frac{\cancel{a}(1-r^6)}{\cancel{1-r}} \times \frac{\cancel{1-r}}{\cancel{a}(1-r^3)}$

$= \frac{1-r^6}{1-r^3} = \frac{\cancel{(1-r^3)}(1+r^3)}{\cancel{1-r^3}}$

$\frac{13.23}{8.75} = 1 + r^3$

$\frac{64}{125} = r^3$

$\boxed{r = \frac{4}{5}}$

Q3

3

A geometric series has first term a and common ratio $\sqrt{5}$.

Show that the sum of the first ten terms of the series is equal to $ka(\sqrt{5}+1)$, where k is a positive integer to be determined.

[4]

a, $r = \sqrt{5}$, $n = 10$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_{10} = \frac{a(1-\sqrt{5}^{10})}{1-\sqrt{5}}$$

$$= \frac{a(1-5^5)}{1-\sqrt{5}}$$

$$= \frac{-3124a}{1-\sqrt{5}} \times \frac{1+\sqrt{5}}{1+\sqrt{5}}$$

$$= \frac{-3124a(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$$

$$= \frac{-3124a(1+\sqrt{5})}{1+\cancel{\sqrt{5}}-\cancel{\sqrt{5}}-5}$$

$$\boxed{S_{10} = 781a(1+\sqrt{5})}$$

$k = 781$

Q4a

4a

The first three terms in a geometric series are $(2k+3)$, k, $(k-2)$, where $k<0$ is a constant.

(a) Find the value of k.

[5]

(b) Find the sum of the first 12 terms in this series.

[3]

a) r is constant

$$\frac{u_2}{u_1} = \frac{u_3}{u_2}$$

$$\frac{k}{2k+3} = \frac{k-2}{k}$$

$$k^2 = (k-2)(2k+3)$$

$$k^2 = 2k^2+3k-4k-6$$

$$0 = k^2-k-6$$

$$(k-3)(k+2)$$

$$k = \cancel{3},\ -2$$

$k<0$ $\therefore$ $\boxed{k=-2}$

Q4b

4b

The first three terms in a geometric series are $(2k+3), k, (k-2)$, where $k<0$ is a constant.

(a) Find the value of k.

$k=-2$

[5]

(b) Find the sum of the first 12 terms in this series.

[3]

b) Find a and r using $k=-2$ from (a)

$S_n = \frac{a(1-r^n)}{1-r}$

$a = 2k+3 = 2(-2)+3 = -1$

$r = \frac{k}{2k+3} = \frac{k}{a} = \frac{-2}{-1} = 2$

$n = 12$

$S_{12} = \frac{(-1)(1-2^{12})}{1-2} = \boxed{-4095}$

Q5a

5a

The second and fifth terms of a geometric series are 13.44 and 5.67 respectively. The series has first term a and common ratio r.

(a) By first determining the values of a and r, calculate the sum to infinity of the series.

[6]

(b) Calculate the difference between the sum to infinity of the series and the sum of the first 20 terms of the series. Give your answer accurate to 2 decimal places.

[2]

a) $u_2 = 13.44$ $\quad$ $u_5 = 5.67$ $\quad$ $u_n = ar^{n-1}$

$13.44 = ar^{2-1} = ar$ $\quad$ $5.67 = ar^{5-1} = ar^4$ $\quad$ $S_\infty = \frac{a}{1-r}$

$\frac{u_5}{u_2}$ $\quad$ $\frac{5.67}{13.44} = \frac{\cancel{a}r^4}{\cancel{a}r} = r^3$

$\sqrt[3]{\frac{5.67}{13.44}} = r = \frac{3}{4} = 0.75$

$13.44 = a\,0.75$

$a = \frac{13.44}{0.75} = 17.92$

$S_\infty = \frac{17.92}{1-0.75} = \frac{17.92}{.25} = \boxed{71.68}$

Q5b

5b

The second and fifth terms of a geometric series are 13.44 and 5.67 respectively. The series has first term a and common ratio r.

(a) By first determining the values of a and r, calculate the sum to infinity of the series. [6]

$a = 17.92$
$r = 0.75$

$S_\infty = 71.68$

(b) Calculate the difference between the sum to infinity of the series and the sum of the first 20 terms of the series. Give your answer accurate to 2 decimal places. [2]

b) $S_\infty - S_{20}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_{20} = \dfrac{17.92(1-0.75^{20})}{1-0.75}$

$= 71.45...$

$S_\infty - S_{20} = 71.68 - 71.45...$

$= \boxed{0.23}$ (2dp)

Q6a

6a

A geometric progression has first term 9, and the sum of the first three terms of the progression is 19. The common ratio of the progression is r.

(a) Show that $9r^2 + 9r - 10 = 0$. [3]

(b) Find the two possible values of r. [2]

(c) Given that the sum to infinity of the progression exists, find the sum to infinity of the progression. [3]

a) $a = 9$
r
$S_3 = 19$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$n = 3$

$S_3 = 19 = \dfrac{9(1-r^3)}{1-r}$

$19 - 19r = 9 - 9r^3$

$9r^3 - 19r + 10 = 0$

$(r-1)(9r^2 + 9r - 10) = 0$

If $r - 1 = 0$, $r = 1$ and $S_3 = 27$! $\therefore r \neq 1$

$\therefore \boxed{9r^2 + 9r - 10 = 0}$

Q6b

6b

A geometric progression has first term 9, and the sum of the first three terms of the progression is 19. The common ratio of the progression is r.

(a) Show that $9r^2 + 9r - 10 = 0$. [3]

(b) Find the two possible values of r. [2]

(c) Given that the sum to infinity of the progression exists, find the sum to infinity of the progression. [3]

b) $9r^2 + 9r - 10 = 0$

$(3r+5)(3r-2)$

$\boxed{r = -\frac{5}{3}, \frac{2}{3}}$

Q6c

6c

A geometric progression has first term 9, and the sum of the first three terms of the progression is 19. The common ratio of the progression is r.

(a) Show that $9r^2 + 9r - 10 = 0$. [3]

(b) Find the two possible values of r.

out of range

$r = \cancel{-\frac{5}{3}}, \frac{2}{3}$ [2]

(c) Given that the sum to infinity of the progression exists, find the sum to infinity of the progression. [3]

c) $-1 < r < 1$ for the S_∞ to exist.

$S_\infty = \frac{a}{1-r}$

$\therefore r = \frac{2}{3}$

$(a = 9)$

$S_\infty = \frac{9}{1-\frac{2}{3}} = \boxed{27}$

Q7a

7a

The kth term of a geometric progression is given by $u_k = 2401\left(\frac{2}{7}\right)^k$.

Calculate, giving your answers as exact values

(a) The sum to infinity of the progression starting with the seventh term.

[3]

(b) The sum to infinity of the progression whose kth term is given by $v_k = u_{k+4}$, where u_k is defined as above.

[2]

a) $k=1, \quad a = u_1 = 2401\left(\frac{2}{7}\right)^1 = 686$

$r = \frac{2}{7}$

$S_\infty = \frac{a}{1-r}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$$S_{7\to\infty} = S_\infty - S_6$$

$$= \frac{686}{1-\frac{2}{7}} - \frac{686\left(\left(\frac{2}{7}\right)^6 - 1\right)}{\left(\frac{2}{7}-1\right)}$$

$$= \frac{4802}{5} - \frac{47034}{49}$$

$$= \boxed{\frac{128}{245}}$$

Q7b

7b

The kth term of a geometric progression is given by $u_k = 2401\left(\frac{2}{7}\right)^k$.

Calculate, giving your answers as exact values $\quad a = 686, \quad r = \frac{2}{7}$

(a) The sum to infinity of the progression starting with the seventh term.

$S_\infty - S_6 = \frac{4802}{5} - \frac{47034}{49}$

[3]

(b) The sum to infinity of the progression whose kth term is given by $v_k = u_{k+4}$, where u_k is defined as above.

[2]

This new progression is similar to the first progression except the first term is u_5!

b) $S_\infty - S_4$

$S_n = \frac{a(r^n - 1)}{r-1}$

$$S_4 = \frac{686\left(\frac{2}{7}^4 - 1\right)}{\frac{2}{7} - 1}$$

$$= 954$$

$$S_\infty - S_4 = \frac{4802}{5} - 954$$

$$= \boxed{\frac{32}{5}}$$